Two Dimensional Dynamic and Analytical Modeling of a Concrete Slab Track for Stresses-Induced in Subgrade under High Speed Trains

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Abstract The stresses change level is one of the most important factors affecting the development of nonlinear behavior of subsoil materials in railway tracks. In high speed lines, the ground vibrations induced by trains have a significant dynamic effect on the stresses produced in the subsoil and the rise of train speed will generate such strong vibrations that will increase and accelerate the development of the larger stresses in subsoil. In this present paper, for the computational analysis, a dynamic numerical simulation in finite element method and a static analytical method are both attempted to compute stresses and their distribution in two dimensional slab track model and a homogenous half-space respectively. The results of the numerical results are compared with analytical solutions to draw conclusions.

Keywords: Dynamic analysis, Analytical method, Concrete slab track, Stresses in subsoil

1. Introduction

With increasing high-speed trains around the world, predicting the dynamic response resulting from passing train traffic is a big concern for the performance of the track during its service life. The passage of train wheels induces ground vibrations resulting in the acceleration of the increase of stresses in the subgrade soil. It is worth noting that the dynamic loads produce larger subgrade responses than the static loads (Waters and Selig, 1995). Researchers have studied the dynamic response of railway subgrade for many years; most of them (Madshus and Kaynia, 2000; Powrie et al., 2008) reported field measurement data on increase of the vibration level with increasing train speed for soft subgrades of ballasted tracks. Since then, many researchers have started investigating the subgrade dynamics behavior through the analytical methods (Krylov, 1995; Yang et al., 1996). Recently, some researchers used numerical simulation methods, such us the finite element methods and the boundary element method, to investigate the dynamic response of soil under traffic loads (Yang et al., 2009; Lombaert and Degrande, 2001). Besides the dynamic numerical methods, however, various empirical and analytical methods including Boussinesq's solutions have been widely employed in rational design for railway track platform (Esveld, 2001; Indraratna et al., 2011). In this paper, stresses in subgrade induced by train vehicle are first calculated via Boussinesq's solution. Then a 2-dimensional dynamic interaction model of vehicle-track-subgrade is formulated in the finite element analysis. The train speed used in the simulations ranges between 100 km/h and 700 km/h.

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2. Boussinesqu Analytical Method

The first attempt to express the stress distribution within an elastic half space was made by Boussinesq (1885). He applied the theory of homogeneous elastic half space to calculate the stress within the subsoil. Recently, Malek et al. (2002) applied this method to calculate the stress cycle in the subsoil of a railway track. There are three main assumptions in this method. First, the track load is uniformly distributed upon the top of sleeper as a strip-load of width 2b. Second, all of the substructure materials are homogeneous and elastic. Third is that the length of loaded area is infinitely long. As shown in Fig. 1, the distributed load represents the constant pressure P_o applied on a strip area by the passage of a vehicle at constant speed V.

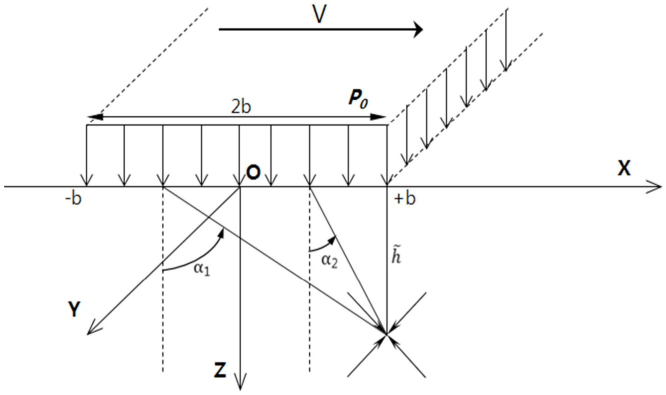

Fig.1 Stress due to strip load on half space

By integrating the elementary Boussinesq's solution on the strip, the vertical stress at any depth of z in Fig. 1 can be obtained as (Poulos, 1991),

$$P_{zi} = P_o f(x_i) \tag{1}$$

In which

$$f(x_i) = \frac{1}{\pi}\left[\alpha_1 - \alpha_2 + \frac{1}{2}\left(\sin 2\alpha_1 - \sin 2\alpha_2\right)\right] \tag{2}$$

The parameters, α_1 and α_2, are defined as:

$$\alpha_1 = \arctan((x_i + b/2)/\tilde{h}) \ \text{ and } \ \alpha_2 = \arctan((x_i - b/2)/\tilde{h}) \tag{3}$$

where $\tilde{h}$ is the length from the loaded area to the observation point. The corresponding expressions of the two first stress invariants defined as:

$$\begin{cases} p = \dfrac{2P_o}{3\pi}(1+\nu)(\alpha_1 - \alpha_2) \\ q = \dfrac{P_o}{\pi}\left[(1-2\nu)^2(\alpha_1 - \alpha_2)^2 + 3\sin^2(\alpha_1 - \alpha_2)\right]^{\frac{1}{2}} \end{cases} \tag{4}$$

where ν is the Poisson's ratio. The pressure distribution method of the moving wheel load in the track system is used to calculate P_0 (Lichtberger, 2005). The concrete slab track system consists of TCL, HSB, reinforced roadbed and subsoil. To employ the above equations, these four layers should be converted into an equivalent single layer. This can be achieved via an empirical equation proposed by Odemark (1949), given by

$$\tilde{h} = \left\{ h_1\left(\frac{E_1}{E_{nl}}\frac{1-\mu_{nl}^2}{1-\mu_1^2}\right)^{1/3} + h_2\left(\frac{E_2}{E_{nl}}\frac{1-\mu_{nl}^2}{1-\mu_2^2}\right)^{1/3} + \ldots\ldots + h_{n-1}\left(\frac{E_{nl-1}}{E_{nl}}\frac{1-\mu_{nl}^2}{1-\mu_{nl-1}^2}\right)^{1/3} \right\} \tag{5}$$

where h_i = thickness of the *i*-th layer, E_i = Young's modulus of the *i*-th layer, ν_i = Poisson's ratio of the *i*-th layer, nl = number of layers. A locomotive train vehicle supported by four wheels is assumed in an elastic half space. As shown in Fig. 2, the superposition method is applied to obtain the maximum stress at an observation point in the subgrade layer.

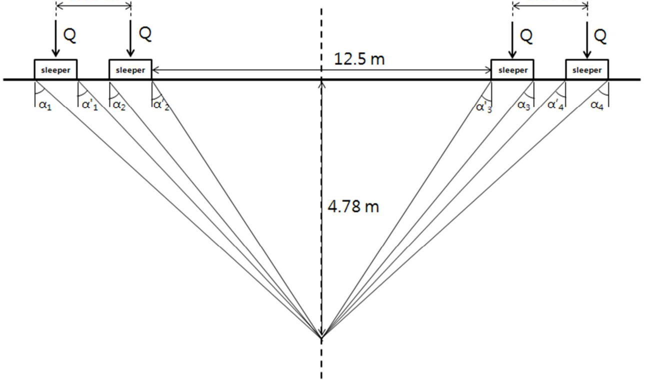

Fig. 2 Simplified 4 wheels loading system for stresses due to strip load on half space

Calculation of the subgrade stress using Eqs. (4) and (5) requires effective wheel load, Q = 102 kN, applied pressure under sleeper surface, P_0 = 300 kPa, sleeper width, b = 0.3 m, and equivalent depth, $\widetilde{h}$ = 4.78 m.

2. Finite Element Model and Evaluation of parameters

The track considered herein is a high-speed line segment of Osongo-Mokpo in Korea. Fig. 3 illustrates the concrete slab with UIC 60 rails supported every 0.65 m by monoblock concrete sleepers, the track concrete layer (TCL) and hydraulically stabilized base (HSB). The reinforced roadbed including sub-ballast and crushed stone layers is located beneath the concrete slab. Dimensions and materials properties of the track are summarized in Table 1 and 2. To perform a dynamic analysis, unfavorable reflections stress waves caused by boundaries were removed by using infinite elements. The Korean Higih Speed Train “Hemu” locomotive is considered and simply modeled as a moving mass oscillator system which is implemented in ABAQUS (2008). Table 2 summarizes the properties of the sprung axle mass, the primary suspension and the static wheel load.

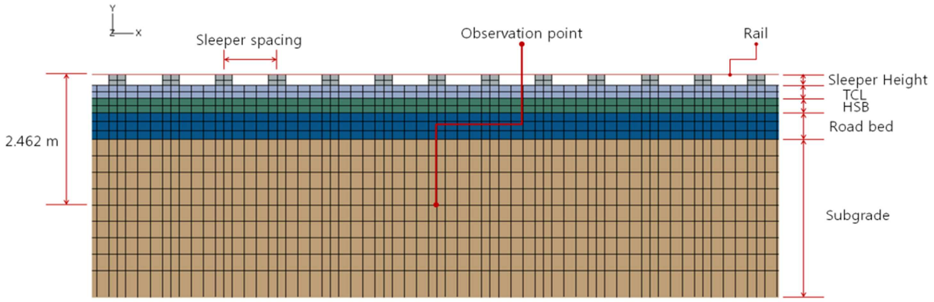

Fig. 3 Slab track model in 2 D Finite Element Method

Table 1 Track Dimensions

Track model length	150m
TCL	0.24m
HSB	0.284m
Reinforced roadbed	0.5m
Subgrade	2.976m
Sleepers height	0.2m
Sleepers length	0.3
Pad thickness	0.01m
Sleepers spacing	0.65m

Table 2 Materials properties and Vehicle parameters in FEM

Material	ρ (t/m3)	E(GPa)	ν	K (kN/m)	C (kNs/m)	Mass (kg)	Wheel Load (kN)
Sprung	-	-	-	1595	15	5750	85
Rail	7.8	210	0.3				
Pad	1.28	6.926e-3	0.49				
Sleeper	2.3	29.1	0.2				
TCL	2.3	34.0	0.2				
HSB	2.3	12.9	0.2				
Reinforced Roadbed	2.0	1.8e-1	0.2				
Subgrade	2.0	6.0e-2	0.3				

3. Results and Discussions

The static analytical response of stress cycle at the observation point is compared to that obtained by numerical static simulation. In terms of the maximum values of Δp and Δq, both analytical and numerical values are very close, as shown in Fig. 4. Fig. 5 shows plotted dynamic stress cycles curves for subgrade. This reveals that when the train speed ranges between 100 and 200 km/h the stress cycles take almost linear form with less area in loops while for further speeds the elliptical form is predominating especially for 600 and 700 km/h with larger area in formed loops. Comparing both analytical results and dynamic results it is revealed that for the speed ranging between 100 and 300 km/h the deviator stresses are highly 5 times while mean stresses are 2.5 times of those of analytical and numerical static results. For the speeds ranging between 400 and 500 km/h, the deviator stresses are 7 times and 3 times for mean stresses, whereas for further increased speeds between 600 and 700 km/h the deviator stresses become 8 times and 4.5 times for mean stresses more than of those of static results.

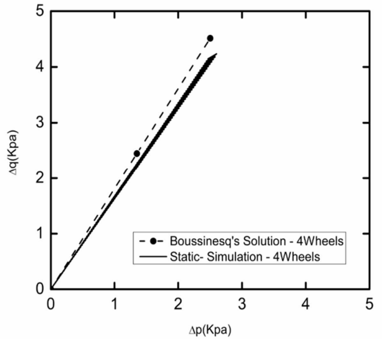

Fig. 4 Stress cycles computed in two methods

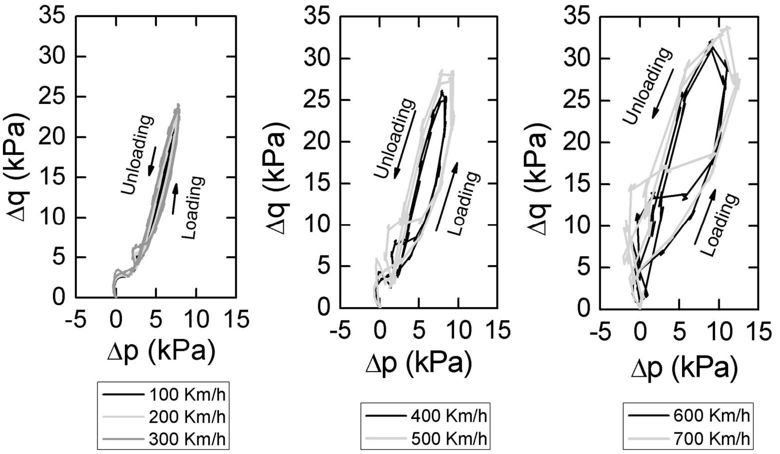

Fig. 5 Dynamic stress change cycles

4. Conclusions

In this analysis, a linear elastic model was performed to investigate the dynamic response of subgrade layer through stress cycles. Of great interest is the influence of the train dynamic velocity on the magnitude of the induced stresses in the subgrade, the analytical response of a homogeneous half space under a constant strip load was introduced to compare the results. The predicted vertical stresses were analyzed and the following conclusions are drawn. The stresses cycles computed through analytical half space response are compared with those obtained from numerical quasi static, and a good agreement between them is revealed, while the induced dynamic stresses response is much higher than of those of both static response.

Acknowledgement

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